Career Makers

It's all about believing

Topic:-Expected Question for M-3(PTU) 2012 nov.

Note 1: Do learn the expansions of e^x, e^{-x}, log (1+x), log (1-x), sinx, cosx, $(1-x)^{-1}$, $(1-x)^{-2}$

Note 2: No two marks questions in these questions:

Functions of Complex Variables (Weightage 18-20 marks)

Q1. Every analytic function f(z)=u+iv defines two families of curves u(x,y)=c_1 and v(x,y)=c_2, which form an orthogonal system.

Q2. An analytic function with constant modulus is constant.

Q3. If u(x,y)=x^2-y^2 and $v(x,y)=\frac{y}{x^2+y^2}$, prove that both u and v satisfy Laplace's equation but are not harmonic conjugates.

Q4. Show that the function defined by f(z)= $\sqrt{|xy|}$ is not regular at the origin, although Cauchy-Riemann equations are satisfied.

Q5. Determine the analytic function whose real part is e^{2x}(xcos2y-ysin2y).

Q6. If $u+v=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$ and f(z)=u+iv is an analytic function of z=x+iy, find f(z) in terms of z.

Q7. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

Q8. If f(z) is analytic within and on a closed curve C and a is any point within C, $f(a)=\frac{1}{2\pi}\oint_C \frac{f(z)}{z-a}dz$.

Q9. Find the Laurent's series expansion of the function $f(z)=\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3<|z+2|<5.

Q10. Evaluate $\int_0^{2\pi}\frac{d\theta}{1-2a\sin\theta+a^2}, 0<a<1$.

Q11. Use complex integration method to prove that $\int_0^{2\pi}\frac{\sin^2\theta}{a+b\cos\theta}d\theta=\frac{2\pi}{b^2}\left(a-\sqrt{a^2-b^2}\right)$, where 0<b<a.

Q12. Use contour integration method to evaluate the following integral $\int_0^{\pi}\frac{a\ d\theta}{a^2+\sin^2\theta}d\theta$.

Q13. Using contour integration, prove that $\int_{-\infty}^{\infty}\frac{dx}{(1+x^2)^2}=\frac{\pi}{2}$. Hence or otherwise evaluate $\int_0^{\infty}\frac{dx}{(1+x^2)^2}$.

Q14. Apply calculus of residues to prove that $\int_0^{\infty}\frac{dx}{(x^2+a^2)^3}=\frac{\pi}{4a^3}; a>0$.

Q15. Apply calculus of residues to prove that $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+a^2)(x^2+b^2)}=\frac{\pi}{a+b}; a>0, b>0$

Fourier series (Weightage 10-14 Marks)

Q16. Obtain the Fourier series for the function f (x) = x^2, $-\pi \le x \le \pi$. Sketch the graph of f (x). Hence show that:

(a) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\dots\dots+\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$. (b) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}\dots\dots\dots=\frac{\pi^2}{12}$ (c) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}\dots\dots\dots=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$

Q17. Obtain Fourier series for the function f (x) given by f (x) = $\begin{cases}1+\frac{2x}{\pi}, & -\pi\le x\le 0\\ 1-\frac{2x}{\pi}, & 0\le x\le\pi\end{cases}$, Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}\dots\dots\dots=\frac{\pi^2}{8}$.

Q18. Find the Fourier series of f (x)=x^2; $0 \le x \le \pi$, =$-x^2$;$-\pi \le x \le 0$.

Q19. Obtain Fourier series for the function: f (x) = $\begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ and Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}.........=\frac{\pi^2}{8}$.

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Laplace transforms (Weightage 12-14 Marks)

Q22. Evaluate $\int_0^t \frac{e^{-at}-e^{-bt}}{t}dt$.

Q23. The currents I_1 and I_2 in mesh are given by differential equations. $\frac{dI_1}{dt}-wI_2=a\cos pt$, $\frac{dI_2}{dt}+wI_1=a\sin pt$ with $I_1=I_2=0$ at t=0. Using Laplace transformation technique, find the currents $I_1(t)$ and $I_2(t)$.

Q24. Using convolution theorem find the inverse of $\frac{16}{(s-2)(s+2)^2}$.

Q25. Solve using Laplace transforms $y''+2y'+5y=e^{-t}$ sint, y (0) =0, $y'(0)$ =1.

Q26. Evaluate: $\int_0^\infty e^{-t}\ \frac{\sin^2 t}{t}dt$.

Q27. Evaluate: $\log\frac{p(p+4)}{p^2+4}$.

Q28. Using Laplace transformation, Solve the differential equation $\frac{d^2x}{dt^2}$ +9x=cos2t, if x (0) =1, x= -1.

Partial Differential Equations (Weightage 10-12 Marks)

Q29. Solve (y^2+z^2) p-xyq+zx+=0.

Q30. Form a differential equation by eliminating the arbitrary function F $(xy,z^2, x+y+z)$ =0.

Q31. Solve: $(D^2-a^2D'^2)$ z=x.

Q32. Solve the linear partial differential equation: $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}$=ycosx.

Q33. Solve: $z(x+y)p+z(x-y)q=x^2+y^2$.

Q34. Solve the following equations: (y+z)p+(z+x)q=x+y.

Q35. A tightly stretched flexible string has its ends fixed at x=0 and x=1. At time t=0 the string is given a shape defined by F (x) = μ x (*l*-x), μ is a constant and then released. Find the displacement y(x,t) of any point x of the string at any time t>0.

Q36. Find the temperature u(x, t) in a homogenous bar of heat conducting material of length *l* cm with its and kept at zero temperature and initial temperature is given by $\frac{dx(L-x)}{L^2}$.

Q37. If a string of length *l* is initially at rest in equilibrium position and each of its points is given the velocity.

Q38. Discuss all possible solutions of the differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.

Q39. A homogenous rod of conducting material of length 100 cm has its ends kept at zero temperature ture and the temperature initially is u(x,0)= $\begin{cases} x, & 0 \le x \le 50 \\ 100 - x, & 50 \le x \le 100 \end{cases}$. Find the temperature u(x,t) at any time.

Q40. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form u= $a\sin\frac{\pi x}{l}$ from which it is released at a time t=0. Show that the displacement of any point at a distance x from one end time t is given by: $u(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$.

Q41. If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b\sin^3\frac{\pi x}{l}$, find the displacement y(x,t).

Special Functions (Weightage 18-20 Marks)

Q42. Recurrence Relation for Bessel functions (Sure).
Q43. Recurrence Relation for Legendre Polynomials (Sure)
Q44. Orthogonality of Legendre Polynomials.
Q45. Orthogonality of Bessel functions.

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Q46. Assume 'n' an integer and J_n (x) a solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0;$ Show that the other independent solution Y_n(x) is given by Y_n(x)= $J_n(x)\int\frac{dx}{xJ_n^2(x)}$.

Q47. Evaluate $\int x^2 J_1(x)dx$.

Q48. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\pi x}J_{1/2}(2x)dx = 1$.

Q49. Solve in series: $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.